Period 1, May 21, 2025

Theorem 8.1 Integration by Parts

If u and v are functions of x and have continuous derivatives, then

$$\int u\,dv = uv - \int v\,du$$

Example:

$\int x e^x \, dx$

$\int u \, dV$

$u = x$ $\int dV = \int e^x dx$

$du = dx$ $V = e^x$

$\int u\,dV = x \cdot e^x - \int e^x dx$

$\int x e^x dx = xe^x - e^x + C$

$y = xe^x - e^x$

$\frac{dy}{dx} = 1 \cdot \cancel{e^x} + x \cdot e^x - \cancel{e^x} = xe^x$

Example:

$\int x^2 \sin x \, dx$

$u = x^2$ $dV = \sin x \, dx$

$du = 2x\,dx$ $V = -\cos x$

$\int x^2 \sin x \, dx = x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x \, dx$

$\int x^2 \sin x \, dx = -x^2 \cos x + \int 2x \cos x \, dx$

$u = 2x$ $dV = \cos x \, dx$

$du = 2dx$ $V = \sin x$

$2x \sin x - \int \sin x \cdot 2 dx$

$\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x - 2(-\cos x) + C$

$\cancel{2x \cos x} + -x^2 \cdot -\sin x + \cancel{2 \sin x} + \cancel{2x \cos x} + -\cancel{2 \sin} x = x^2 \sin x$

$$\int arc \sin x \, dx = x\arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} \, dx$$

$U = \arcsin x \qquad dV = dx$

$dU = \frac{1}{\sqrt{1-x^2}} dx \qquad V = x$

$$x\arcsin x - \int \frac{x}{\sqrt{1-x^2}} \, dx$$

$u = 1 - x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

$$x\arcsin x + \int \frac{\cancel{x}}{\sqrt{u}} \cdot \frac{dU}{+2\cancel{x}}$$

$$x\arcsin x + \frac{1}{2}\int U^{-\frac{1}{2}} \, dU$$

$$x\arcsin x + \frac{\cancel{1}}{\cancel{2}}\left(\cdot \frac{\cancel{2}}{1} \cdot U^{-\frac{1}{2}+1=\frac{1}{2}}\right) + C$$

$$x\arcsin x + \sqrt{1-x^2} + C$$

Example:

$$\int e^x \sin x \, dx = \frac{-e^x \cos x + e^x \sin x}{2}$$

$u = e^x \qquad dv = \sin x \, dx$

$du = e^x \, dx \qquad v = -\cos x$

$$\int e^x \sin x \, dx = -e^x \cos x + \int +\cos x \cdot e^x \, dx$$

$u = e^x \qquad dv = \cos x \, dx$

$du = e^x \, dx \qquad v = \sin x$

$$\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \cancel{\int \sin x \, e^x \, dx}$$

$$+\int e^x \sin x \, dx \qquad \cancel{+\int \sin x \, e^x \, dx}$$

$$\frac{\cancel{2}\int e^x \sin x \, dx}{\cancel{2}} = \frac{-e^x \cos x + e^x \sin x}{2}$$

$$\int x^3 \sin 2x \, dx = -\left(\tfrac{1}{2}\cos 2x\right)x^3 - \int -\tfrac{1}{2}\cos 2x \cdot 3x^2 \, dx$$

$u = x^3 \qquad dv = \sin 2x \, dx$

$du = 3x^2 \, dx \qquad v = -\frac{1}{2}\cos 2x$

$$= -\frac{x^3}{2}\cos 2x + \frac{3}{2}\int x^2 \cos 2x \, dx$$

$u = x^2 \qquad dv = \cos 2x \, dx$

$du = 2x \, dx \qquad v = \frac{1}{2}\sin 2x$

$$-\frac{x^3}{2}\cos 2x + \frac{3}{2}\left[\frac{1}{2}\sin 2x \cdot x^2 - \int \left(\tfrac{1}{2}\sin 2x\right) \cdot 2x \, dx\right]$$

$$-\frac{x^3}{2}\cos 2x + \frac{3x^2}{4}\sin 2x - \frac{3}{2}\int x \sin 2x \, dx$$

$u = x \qquad dv = \sin 2x \, dx$

$du = dx \qquad v = -\frac{1}{2}\cos 2x$

$$-\frac{x^3}{2}\cos 2x + \frac{3x^2}{4}\sin 2x - \frac{3}{2}\left[-\frac{x}{2}\cos 2x + \int +\frac{1}{2}\cos 2x \cdot dx\right]$$

$$-\frac{x^3}{2}\cos 2x + \frac{3x^2}{4}\sin 2x + \frac{3x}{4}\cos 2x - \frac{3}{2}\cdot\frac{1}{4}\sin 2x + C$$

$\int x^3 \sin 2x \, dx$ **Let $u = x^3$** **Let $dv = \sin 2x \, dx$**

$+$	x^3	$\sin 2x dx$
$-$	$3x^2 dx$	$-\frac{1}{2}\cos 2x$
$+$	$6x dx$	$-\frac{1}{4}\sin 2x$
$-$	6	$+\frac{1}{8}\cos 2x$
		$\frac{1}{16}\sin 2x$

$$-\frac{1}{2}\cos 2x \cdot x^3 - \left(-\frac{3}{4}x^2 \sin 2x\right) + \frac{6x}{8}\cos 2x - 6 \cdot \frac{1}{16}\sin 2x$$

Same

$$-\frac{x^3}{2}\cos 2x + \frac{3x^2}{4}\sin 2x + \frac{3x}{4}\cos 2x - \frac{3}{8}\sin 2x + C$$

1. $\int (4x + 7)e^x \, dx$

$u = 4x+7$ $\quad dv = e^x dx$

$du = 4dx$ $\quad v = e^x$

$$(4x+7)e^x - \int e^x \cdot 4dx$$

$$4xe^x + 7e^x - 4e^x + C$$

$$4xe^x + 3e^x + C$$

5. $\int e^{2x}\cos x\,dx = e^{2x}\sin x - \int \sin x \cdot 2e^{2x}\,dx$

$u = e^{2x}$ $\quad dv = \cos x\,dx$

$du = 2e^{2x}\,dx$ $\quad v = \sin x$

$= e^{2x}\sin x - 2\int \sin x\, e^{2x}\,dx$

$u = e^{2x}$ $\quad dv = \sin x\,dx$

$du = 2e^{2x}\,dx$ $\quad v = -\cos x$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - 2\left[-e^{2x}\cos x + \int +\cos x \cdot 2e^{2x}\,dx\right]$$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x + 2e^{2x}\cos x - 4\int \cos x \cdot e^{2x}\,dx$$

$+4\int e^{2x}\cos x\,dx$ $\qquad +4\int \cos x \cdot e^{2x}\,dx$

$$\frac{\cancel{5}\int e^{2x}\cos x\,dx}{\cancel{5}} = \frac{e^{2x}\sin x + 2e^{2x}\cos x}{5}$$

$$\int (x^3 + 2x - 1)\cos 4x\ dx$$

	u	dv
$+$	x^3+2x-1	$\cos 4x\,dx$
$-$	$3x^2+2$	$\frac{1}{4}\sin 4x$
$+$	$6x$	$-\frac{1}{16}\cos 4x$
$-$	6	$-\frac{1}{64}\sin 4x$
		$+\frac{1}{256}\cos 4x$

$$(x^3+2x-1)\cdot\frac{1}{4}\sin 4x - (3x^2+2)\left(-\frac{1}{16}\cos 4x\right) + 6x\left(-\frac{1}{64}\sin 4x\right) - 6\left(\frac{1}{256}\cos 4x\right)$$